Motion and Forces in a Gravitational Field: Set 4

Set	Problem	Solution
4	1	One of the masses need to be HUGE to have any noticeable gravitational attraction between them, huge means greater than 10^{20} kg
	2a	No, 8km when compared to the radius of the earth is insignificant.
	2b	A very deep mine is still a very small change in r, compared to the radius of the earth. So you would measure no change in your weight. If you consider the earth to be a perfect sphere then your weight would decrease as you moved towards the centre of the earth because the mass enclosed in the R would be less.
	2c	The density of the ground varies significantly based on the ore deposits etc in an area. The Earth is not 100% spherical, it 'bulges' which generates slight changes in 'g'.
	3	The weight of a freely falling object is equal to its mass ×g. We feel 'weightless' when we fall freely accelerating at 9.8 m s^{-2} because if we are in a lift, (for example) we would no longer feel the reaction force from the floor pushing up on us.
	4	Because the force exerted on each kilogram is the same, so from Neewton's 2nd law, a= F/m, if m=4, the force = 9.8 × 4, but a = 9.8 m s^{-2}
	5	Gravitational force is proportional to $1/r^2$ so as r halves gravitational force increases by a factor of 4.
	6	$M_E = \frac{r_E^2 \times g}{G} = \frac{(6.37 \times 10^6\ m)^2 \times 10\ m\,s^{-2}}{6.67 \times 10^{-11}\,N\,m^2 kg^{-1}}$ $=6 \times 10^{24}$ kg
	7	$F = G\frac{M_1 M_2}{d^2} = 6.67 \times 10^{-11}\,Nm^2kg^{-1} \times \frac{100\,kg \times 100\,kg}{(622 \times 10^{-3} m)^2}$ 1.72×10^{-6} N
	8a	Since F ? $1/d^2$.If F is to fall by 0.5, then d should increase by $\sqrt{2}$ .So$\sqrt{2}$ -1 =0.414. $0.414 \times R_E = 64 \times 10^6$ m
	8b	$g = \frac{GM_E}{(R_E + 610 \times 10^3)^2}$ = 8.16 m s^{-2} toward the Earth
	8c	$G\frac{M_E M_S}{r^2} = \frac{M_S v^2}{r}$ which leads to $v^2 = G\frac{M_E}{r} = 7.55 \times 10^3$ m s^{-1}
	9	$d^2 = G\frac{M_1 M_2}{F} = 6.67 \times 10^{-11}\,Nm^2kg^{-1} \times \frac{5.98 \times 10^{24}\,kg \times 7.34 \times 10^{22}\,kg}{2.03 \times 10^{20}\,N}$ $d = 3.80 \times 10^8$ m
	10	$g \propto M_{planet}$ and $g \propto \frac{1}{R_{planet}^2}$ therefore $g_{Neptune} = 16.6/(3.89)^2 \times g_{Earth} = 1.1 \times g_{Earth}$
	11a	Force is constant, the acceleration and velocity change direction but the magnitude of the speed stays the same.
	11b	2.38×10^{20} N toward the Sun
	12	$T^2 \propto R^3$, therefore the higher the orbit the larger the period
	13a	It would no longer be in a stable orbit and could fall to the Earth.
	13b	They only appear to be stationary. They are moving with the same period of revolution as the point on the Earth's surface.
	14a	That is the way the Earth is rotating.
	14b	Launching at the equator puts a satellite straight into a geostationary orbit , launching from another point would require rocket fuel to get the satellite into the correct position.
	15	Drop it in the opposite direction to the motion of the space station.
	16	As R decreases F increases, therefore acceleration increases and so does velocity
	17	The space station is a satellite and is in free fall around the Earth. No normal force is supplied by the space station to the astronauts.
	18	$v^2 = \frac{GM_E}{r}$,subsitite for v^2 $\frac{4\pi^2 r^2}{T^2} = \frac{GM_E}{r}$ $T = \sqrt{\frac{4\pi^2 r^3}{GM_E}} = \sqrt{\frac{4\pi^2(550 \times 10^3 + 6.37 \times 10^6)^3}{G \times 5.98 \times 10^{24}}}$ 5.74×10^3 s (1.59 hours)

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4	19a	$v = \frac{2\pi r}{t} = \frac{2\pi \times 6.71 \times 10^8 m}{3.07 \times 10^5 s}$ 1.37×10^4 m s^{-1}
	19b	$M_1 = \frac{v^2 r}{G} = \frac{(1.37 \times 10^4)^2 \times 6.71 \times 10^8}{G}$ 1.90×10^{27} kg
	20	$M_E = \frac{4\pi^2 r^3}{Gt^2} = \frac{4\pi^2(6.37 \times 10^6 + 2.02 \times 10^7)^3}{G \times (12 \times 3600)^2}$ 5.97×10^{24} kg
	21	If satellite is geostationary then period is 24 hours From $M_E = \frac{4\pi^2 r^3}{Gt^2}$, $r = \sqrt[3]{\frac{M_E G t^2}{4\pi^2}}$ $r = \sqrt[3]{\frac{M_E \times G \times (24 \times 3600)^2}{4\pi^2}}$ 3.59×10^7 m
	22	$r = \sqrt[3]{\frac{M_1 G t^2}{4\pi^2}}$ Using the equation above and cancelling everything that is similar Ratio Titan:Moon = $\sqrt[3]{108 \times 14^2} \div \sqrt[3]{27.3^2}$ 3.05 × (radius of Moon's orbit around Earth)
	23a	$F = G\frac{M_1 M_2}{d^2}$ At perihelion $F = G\frac{1.99 \times 10^{30} \times 3.30 \times 10^{23}}{(4.6 \times 10^{10})^2}$ 2.07×10^{22} N; Similarly at aphelion 9.20×10^{21} N
	23b	Velocity also changes $v = \sqrt{\frac{Fr}{m}}$ At perihelion $v = \sqrt{\frac{2.07 \times 10^{22} \times 4.6 \times 10^{10}}{3.30 \times 10^{23}}}$ 5.37×10^4 m s^{-1}; Similarly at aphelion 4.39×10^4 m s^{-1}
	23c	Total energy stays constant, as the kinetic energy decreases (with increasing r) then gravitational potential energy increases (with increasing r).